10-6 Exponential Growth and Decay

Depreciation $y = a(1 - r)^t$

Decay $y = ae^{-kt}$

EX: A cup of coffee contains 130 milligrams of caffeine. If caffeine is eliminated from the body at a rate of 11% per hour, how long will it take for half of this caffeine to be eliminated from a person's body?

$y = a(1-r)^t$

$65 = 130(1-.11)^t$

$\frac{1}{2} = .89^t$

$\log\frac{1}{2} = t\log .89$

$\frac{\log\frac{1}{2}}{\log .89} = t$

$5.9\text{ hrs} = t$

How long will it take for 90% of this caffeine to be eliminated from a person's body?

$13 = 130(.89)^t$

$.1 = .89^t$

$\frac{\log .1}{\log .89} = \frac{t\log .89}{\log .89}$

$19.8\text{ hrs} = t$

EX: The half-life of a radioactive substance is the time it takes for half of the atoms of the substance to become disintegrated. All life on Earth contains the radioactive element Carbon-14, which decays continuously at a fixed rate. The half-life of Carbon-14 is 5760 years; that is, every 5760 years, half of a mass of Carbon-14 decays away.

What is the constant for Carbon-14?

$y = ae^{-kt}$

$\frac{1}{2} = 1e^{-k5760}$

$\frac{1}{2} = e^{-5760k}$

$\ln\frac{1}{2} = -5760k$

$.00012 = k$

A paleontologist examining the bones of a wooly mammoth estimates that they contain only 3% as much Carbon-14 as they would have contained when the animal was alive. How long ago did the mammoth die?

$y = ae^{-kt}$

$y = ae^{-.00012t}$

$3 = 100e^{-.00012t}$

$.03 = e^{-.00012t}$

$\ln .03 = -.00012t$

$29{,}221\text{ yrs} = t$

EX: The half-life of Sodium-22 is 2.6 years. What is the constant for Sodium-22?

A geologist examining a meteorite estimates that it contains only about 10% as much Sodium-22 as it would have contained when it reached Earth's surface. How long ago did the meteorite reach the surface of the Earth?

Appreciation

$y = a\ (1 + r)^t$

$y = a(1 \pm r)^t$

$y = a e^{\pm kt}$

Growth

$y = ae^{kt}$

EX: In 1910 the population of a city was 120,000. Since then, the population has increased by exactly 1.5% per year. If the population continues to grow at this rate, what will the population be in the year 2010?

$y = a(1+r)^t$

$y = 120000(1+.015)^{100}$

$= 531,845$

EX: The city of Raleigh, North Carolina grew from a population of 212,000 in 1990 to a population of 259,000 in 1998. Write and exponential growth equation in the form:
$y = ae^{kt}$ where t is the number of years after 1990

Predict the population of Raleigh in 2010.

EX: As of 2000, China (the most populous country) had an estimated population of 1.26 billion people. India (the second most populous country) had an estimated 1.01 billion. The populations of India and China can be modeled by:

$I(t) = 1.01e^{.015t}$ and $C(t) = 1.26e^{.0009t}$

When will India's population be more than China's?

$1.01e^{.015t} \geq 1.26e^{.0009t}$

$e^{.015t} \geq 1.25e^{.0009t}$

$.015t \geq \ln(1.25e^{.0009t})$

$\ln 1.25 + \ln e^{.0009t}$

$.015t \geq \ln 1.25 + .0009t$

$.0141t \geq \ln 1.25$

$t \geq 15.83$ yrs

EX: As of 2000, Nigeria had an estimated population of 127 million people and the US had an estimated population of 278 million people. The populations of Nigeria and the US can be modeled by :

$N(t) = 127e^{.026t}$ and $U(t) = 278e^{.009t}$

When will the population of Nigeria be more than the population of the US?

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